

APPENDIX A: GROUPS OF SMALL ORDER

1 = trivial group

$$1 \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

2.1 $\langle A|A^2 \rangle = C_2$

$$\begin{array}{c} 1 \quad 2 \\ 1 \end{array} \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 2 & 1 \\ \hline \end{array}$$

3 $\langle A|A^3 \rangle = C_3$

$$\begin{array}{c} 1 \quad 2 \quad 3 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 2 & 3 & 1 \\ \hline 3 & 1 & 2 \\ \hline \end{array}$$

4.1 $\langle A|A^4 \rangle = C_4$

$$\begin{array}{c} 1 \quad 2 \quad 3 \quad 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 4 \\ \hline 2 & 3 & 4 & 1 \\ \hline 3 & 4 & 1 & 2 \\ \hline 4 & 1 & 2 & 3 \\ \hline \end{array}$$

$$4.2 = \langle \mathbf{A}, \mathbf{B} | \mathbf{A}^2, \mathbf{B}^2, \mathbf{AB=BA} \rangle = \mathbf{V}_4$$

$$= \mathbf{C}_2 \times \mathbf{C}_2$$

	1	2	3	4
1	1	2	3	4
2	2	1	4	3
3	3	4	1	2
4	4	3	2	1

$$5 = \langle \mathbf{A} | \mathbf{A}^5 \rangle = \mathbf{C}_5$$

	1	2	3	4	5
1	1	2	3	4	5
2	2	3	4	5	1
3	3	4	5	1	2
4	4	5	1	2	3
5	5	1	2	3	4

$$6.1 \langle \mathbf{A} | \mathbf{A}^6 \rangle = \mathbf{C}_6 = \mathbf{C}_3 \times \mathbf{C}_2$$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	1	2
4	4	5	6	1	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5

$$6.2 \langle A, B \mid A^3, B^2, AB = BA^{-1} \rangle = D_6 = S_3$$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	1	6	4	5
3	3	1	2	5	6	4
4	4	5	6	1	2	3
5	5	6	4	3	1	2
6	6	4	5	2	3	1

$$7 \langle A \mid A^7 \rangle = C_7$$

	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	3	4	5	6	7	1
3	3	4	5	6	7	1	2
4	4	5	6	7	1	2	3
5	5	6	7	1	2	3	4
6	6	7	1	2	3	4	5
7	7	1	2	3	4	5	6

8.1 $\langle A \mid A^8 \rangle = C_8$

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	3	4	5	6	7	8	1
3	3	4	5	6	7	8	1	2
4	4	5	6	7	8	1	2	3
5	5	6	7	8	1	2	3	4
6	6	7	8	1	2	3	4	5
7	7	8	1	2	3	4	5	6
8	8	1	2	3	4	5	6	7

8.2 $\langle A, B \mid A^4, B^2, AB = BA \rangle = C_4 \times C_2$

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	3	4	1	6	7	8	5
3	3	4	1	2	7	8	5	6
4	4	1	2	3	8	5	6	7
5	5	6	7	8	1	2	3	4
6	6	7	8	5	2	3	4	1
7	7	8	5	6	3	4	1	2
8	8	5	6	7	4	1	2	3

$$8.3 \langle A, B, C \mid A^2, B^2, C^2, AB = BA, AC = CA, BC = CB \rangle = C_2 \times C_2 \times C_2$$

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	1	4	3	6	5	8	7
3	3	4	1	2	7	8	5	6
4	4	3	2	1	8	7	6	5
5	5	6	7	8	1	2	3	4
6	6	5	8	7	2	1	4	3
7	7	8	5	6	3	4	1	2
8	8	7	6	5	4	3	2	1

$$8.4 \langle A, B \mid A^4, B^2, AB = BA^{-1} \rangle = D_8$$

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	3	4	1	8	5	6	7
3	3	4	1	2	7	8	5	6
4	4	1	2	3	6	7	8	5
5	5	6	7	8	1	2	3	4
6	6	7	8	5	4	1	2	3
7	7	8	5	6	3	4	1	2
8	8	5	6	7	2	3	4	1

$$8.5 \langle A, B \mid A^4, B^2 = A^2, AB = BA^{-1} \rangle = Q_8$$

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	3	4	1	8	5	6	7
3	3	4	1	2	7	8	5	6
4	4	1	2	3	6	7	8	5
5	5	6	7	8	3	4	1	2
6	6	7	8	5	2	3	4	1
7	7	8	5	6	1	2	3	4
8	8	5	6	7	4	1	2	3

$$9.1 \langle A \mid A^9 \rangle = C_9$$

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	3	4	5	6	7	8	9	1
3	3	4	5	6	7	8	9	1	2
4	4	5	6	7	8	9	1	2	3
5	5	6	7	8	9	1	2	3	4
6	6	7	8	9	1	2	3	4	5
7	7	8	9	1	2	3	4	5	6
8	8	9	1	2	3	4	5	6	7
9	9	1	2	3	4	5	6	7	8

$$9.2 \langle A, B \mid A^3, B^3, AB = BA \rangle = C_3 \times C_3$$

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	3	1	5	6	4	8	9	7
3	3	1	2	6	4	5	9	7	8
4	4	5	6	7	8	9	1	2	3
5	5	6	4	8	9	7	2	3	1
6	6	4	5	9	7	8	3	1	2
7	7	8	9	1	2	3	4	5	6
8	8	9	7	2	3	1	5	6	4
9	9	7	8	3	1	2	6	4	5

$$10.1 \langle A \mid A^{10} \rangle = C_{10} = C_5 \times C_2$$

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	1
3	3	4	5	6	7	8	9	10	1	2
4	4	5	6	7	8	9	10	1	2	3
5	5	6	7	8	9	10	1	2	3	4
6	6	7	8	9	10	1	2	3	4	5
7	7	8	9	10	1	2	3	4	5	6
8	8	9	10	1	2	3	4	5	6	7
9	9	10	1	2	3	4	5	6	7	8
10	10	1	2	3	4	5	6	7	8	9

10.2 $\langle A, B \mid A^5, B^2, AB = BA^{-1} \rangle = D_{10}$

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	1	10	6	7	8	9
3	3	4	5	1	2	9	10	6	7	8
4	4	5	1	2	3	8	9	10	6	7
5	5	1	2	3	4	7	8	9	10	6
6	6	7	8	9	10	1	2	3	4	5
7	7	8	9	10	6	5	1	2	3	4
8	8	9	10	6	7	4	5	1	2	3
9	9	10	6	7	8	3	4	5	1	2
10	10	6	7	8	9	2	3	4	5	1

NUMBER OF GROUPS OF EACH ORDER UP TO 60

1	1	11	1	21	2	31	1	41	1	51	1
2	1	12	5	22	2	32	51	42	6	52	5
3	1	13	1	23	1	33	1	43	1	53	1
4	2	14	2	24	15	34	2	44	4	54	15
5	1	15	1	25	2	35	1	45	2	55	2
6	2	16	15	26	2	36	14	46	2	56	13
7	1	17	1	27	5	37	1	47	1	57	2
8	5	18	5	28	4	38	2	48	52	58	2
9	2	19	1	29	1	39	2	49	2	59	1
10	2	20	5	30	4	40	14	50	5	60	13